Instructions: Try these problems from the textbook as practice (answers below).

Problems

15-20. Determine whether or not the vector field is conservative. It it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

15. $\mathbf{F}(x, y, z) = \langle 2xy^3z^2, 3x^2y^2z^2, 2x^2y^3z \rangle$

17. $\mathbf{F}(x, y, z) = \left\langle \ln y, \dfrac{x}{y} + \ln z, \dfrac{y}{z} \right\rangle$

19. $\mathbf{F}(x, y, z) = \langle yz^2e^{xz}, ze^{xz}, xyze^{xz} \rangle$

21. Is there a vector field G on $\mathbb{R}^3$ such that $\operatorname{curl} \mathbf{G} = \langle x \sin y, \cos y, z - xy \rangle$? Explain.

23. Show that any vector field of the form $\mathbf{F}(x, y, z) = \langle f(x), g(y), h(z) \rangle$ where f, g, h are differentiable functions, is irrotational (curl is 0).

Answers

15. $f(x, y, z) = x^2y^3z^2 + K$

17. $f(x, y, z) = x \ln y + y \ln z + K$

19. not conservative

21. no